Model Question Paper
B.Sc Sem IV Math - C-10 Ring Theory.

Short Answer type Questions.

1) Define Ring. Prove that the set M of all $n \times n$ matrices with their elements as real numbers is a non commutative ring with unity, with respect to addition and multiplication of matrices as the two ring compositions.

(2) prove that the intersection of two subrings is a subring.

(3) Define Integral domain. show that the set of numbers of the form $a+b\sqrt{2}$, with a & b as rational numbers is a field.

(4) Define characteristic of a ring. prove that the characteristic of an integral domain is either 0 or a prime number.

(5) Define Ideals. If m is a fixed integer, the set P of integers given by,

$$P = \{xm : x \text{ is an integer}\}$$

is an ideal of the ring R of all integers.

(6) Define Prime Ideal & Maximal Ideal with examples.

(7) Define Homomorphism of rings & Kernel of a ring homomorphism.

Prove that If f is a homomorphism of a ring R into a ring R' with Kernel S, then S is an ideal of R.

(8) Suppose R is a ring, S an ideal of R. Let f be a mapping from R to R/S defined by $f(a) = S+a \ \forall\, a \in R$. Then f is a homomorphism of R onto R/S.

(9) Define Polynomial Ring. Prove that the set $R[x]$ of all polynomials over an arbitrary ring R is a ring with respect to addition and multiplication of Polynomials.

(10) Let S be an ideal of a Commutative ring R. Let a be an element of S such that $x \in S \Rightarrow x = ya$ for some $y \in R$. Then prove that S is a principal ideal of R generated by a.

Long answer type Questions.

1) Define subrings. Prove that the necessary and sufficient conditions for a non-empty subset S of a ring R to be a subring of R are

(i) $a \in S, b \in S \Rightarrow a-b \in S$ (ii) $a \in S, b \in S \Rightarrow ab \in S$.

2) Define Field. Prove that every field is an integral domain.

3) Prove that every finite integral domain is a field.

4) Prove that an arbitrary intersection of left ideals of a ring is a left ideal of the ring.

5) Prove that every homomorphic image of a ring R is isomorphic to some quotient ring.

6) Define Isomorphism of Rings.
If f is an isomorphism of a ring R onto a ring R', then prove that
(i) If R is Commutative ring, then R' is also a Commutative ring.

(ii) If R is without zero divisors, then R' is also without zero divisors.

(7) If D is an Integral domain, then the polynomial ring $D[x]$ is also an integral domain.

(8) Prove that a Commutative ring with unity is a field if it has no proper ideals.